It’s Is All About Connections

by Cathy Draper

My teaching life changed when I entered a Masters degree program that included elementary teachers, centered on math education development. I was a high school teacher by training and was only slightly aware of elementary and middle school pedagogy and the instructional models used in the early grades. As a result of my new experiences, I decided that I wanted to identify and make more classroom-relevant connections at all levels within the instructional program—elementary through high school. I wanted to illustrate the continuum, not just provide occasional examples.

I now add my voice to the chorus of other math educators who say that the algebra content in the upper grades isn’t that different than what students have already learned in the lower grades. The clue is in the connections, patterns, and systems – also known as consistencies. This article describes some of the examples that I have “borrowed” from elementary grades and applied to algebraic topics. For example, skip counting (from first grade) or the two’s tables (often learned in third grade) are both recognizable in the algebraic equation **y = 2x**.

Teaching at the high school level, my algebra classroom presentations started changing at a rapid clip. I looked very closely at elementary lessons and manipulatives for clear and obvious clues for students to recognize the algebraic thinking in what they had already done in their earlier years. I wanted my algebra students to recognize the bridge between what they already knew and the algebra symbolism and to assure them that they already “really” knew it. I, in turn, was learning about the building blocks of elementary mathematics at the same time.

I found that the easiest, as well as most efficient, method for creating an environment in which students can make connections is to use the same models. When students see the *same* models in two or more settings then they are clearly more likely to recognize the connections between the two concepts represented in those settings.

For example, if we use Base 10 blocks and place value mats for place value in early grades then we should also use them in subsequent grades even through middle school grades, if necessary. In this way we can illustrate the greater numerical quantities that reflect the patterns in those upper level topics, such as exponents. Some textbooks represent these models at every level in their series, others give it a try but miss the mark and still others don’t even bother. Since our students (and we as teachers to a certain extent) are somewhat dependant on textbooks for the math content and since we know that students do *not* see the same textbook series throughout the grades, then we need to take charge and make the connections clear.

To illustrate how we can do so, I have listed approximate grade levels for the place value mat with some implications and cautions for teachers.

Approximate grade level and place value mat arrangements

Grade 1 and 2:

TABLE 1

1000 thousands	100 hundreds	10 tens	1 ones

Teaching / learning opportunities: Talk about numbers and instead of 1 through 10, use digits 0 through 9 so that the notion of digit isn’t so strange to these young students later. When we teachers use the same vocabulary for different things, we are creating a confusing environment for these early readers. In my experience they see this duality as confusing, one of many inconsistencies in their arithmetic experiences. We are already using the symbol, "5," as a dual representation for the written word "five" so we are already on shaky reading ground for students to learn about the consistency and patterns in mathematics. If we consider this from the perspective of Piaget's work, we would need to ask at what sequenced learning level can students comprehend multiple representations of the same idea? Also, start highlighting the ones column to show at least two important ideas:

1. These digits are the same ones used in the other columns;
2. These digits are important in the interpretations of reading all numbers.

Grade 3 and 4: larger quantities in groups of three (See Table 2)

Teaching / learning opportunities: Continue highlighting the ones column. Make certain that the repetition of the ones, tens, hundreds, repeats in each group of the three columns. The repetition is easier to comprehend as students learn to regroup while learning about operations. Many students whom I have encountered at these upper levels have never recognized this pattern—and they are good students. I believe the trick is in the instructional emphasis on the patterns. We need to remember that these students may or may not be at Piaget's formal operations stage even though they are older than seven. Piaget focused on the sequence of developmental learning stages; most textbooks organize and sequence topics by grade level in a purely chronological manner.

Grade 5 and 6: larger quantities in groups of three and extending to decimal quantities. (See Table 3)

Teaching / learning opportunities: With the ones column highlighted, the symmetry of the location and the notation is more visually obvious. If students have seen this highlighting all along then it isn't a new idea for them. Learning styles, hemisphere dominance, and multiple intelligences have suggested for years, even decades, the significance of the visual impact in learning.

Grade 7 and 8: Use the same mat and add the next exponent row as part of a pattern for illustrating the number of 0's in the place value chart. (See Table 4)

Teaching / learning opportunities: Upper level teachers will note that the zero exponent dilemma takes care of itself as it occurs in the pattern of the exponents. You can use that other rather abstract explanation involving subtraction of exponents described in most textbooks later, since in my experience, it means very little to the students anyway. A natural extension for this mat is to represent the pattern for both positive and negative exponents and therefore decimal numbers. Also all teachers and students will notice that the line of symmetry happens in the ones column, not at the decimal location. All of the digits occur in the ones column and it is the *location of these digits* that makes the larger quantities. That means 10 is not a digit, so we have digits 0 through 9, not 1 through 10.

TABLE 2

100,000,000	10,000,000	1,000,000	100,000	10,000	1000	100	10	1
Hundred million	*Ten* million	*One* million	*Hundred* thousand	*Ten* thousand	*One* thousand	hundred	ten	one

TABLE 3

1000	100	10	1	$\frac{1}{10}$	$\frac{1}{10 \cdot 10}$	$\frac{1}{10 \cdot 10 \cdot 10}$
thousands	hundreds	tens	ones	or 0.1 one tenth	or 0.01 one hundredth	or 0.001 one thousandth

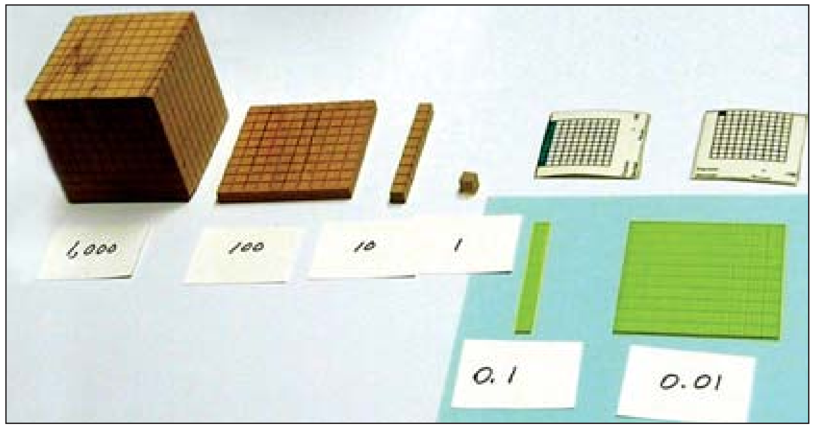
Algebra replaces the 10 with an "x": (See Table 5)

Teaching / learning opportunities: The same place value mat can assist in the transition to algebraic notation substituting an "x" for 10 with the positive and negative exponents, the enigmatic 0 as an exponent, and the negative exponents. This exchange notion was part of the intent in the New Math in the sixties and modular arithmetic for representing the system of the numbering organization.

There are also other materials available on the market, such as Decimal Squares developed by Albert Bennett, that will provide the same square model arrangement for decimals so that educators can use the same or similar design when illustrating multiplication for decimals. Another option for representing decimals that I have tried is the see-through model for Base 10 blocks. The long block is where the symmetry becomes visually obvious.

Because the Decimal Squares do not have a 1/1000 card, I use cards developed by Margaret Smart and Mary Laycock in their 1984 book entitled *Hands on Math* from Activity Resources. The Decimal Squares cards show the square 10, 100, 1000 grid with different shadings very similar to Fraction Bars and are best used when showing how the decimal numbers are representations of 10, 100, and 1000 parts.

Expanding from the place value mat into the algorithmic operations certainly should involve Base 10 blocks (also called Dienes Blocks). The base ten blocks and place value mats are—or should be—a staple of elementary classrooms beginning with place value and continuing with all of the computational processes. Again, some texts accomplish this continuity; others weigh in at differing levels of reliability.



If we use these Base 10 blocks for organizing base 10 notation, then we should also use the same Base 10 blocks to represent the processes involved with operations so that students do not see a "new topic" but an extension of the place value organization. The addition representation with the blocks is presented in most textbooks and teachers often incorporate Base 10 blocks in classroom instruction. However, unless we continue illustrating the operations involving two or more digits with the Base 10 blocks after beginning

TABLE 4

10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
1000	100	10	1	$\frac{1}{10}$ or 0.1	$\frac{1}{100}$ or 0.01	$\frac{1}{1000}$ or 0.001
1010 · 10	10 · 10	10		$\frac{1}{10}$	$\frac{1}{10 \cdot 10}$	$\frac{1}{10 \cdot 10 \cdot 10}$
thousands	hundreds	tens	ones	tenths	hundredths	thousandths

TABLE 5

x^3	x^2	x^1	x^0	x^{-1}	x^{-2}	x^{-3}
$x \cdot x \cdot x$	$x \cdot x$	x	1	$\frac{1}{x}$	$\frac{1}{x^2}$	$\frac{1}{x^3}$
			ones	$\frac{1}{x}$	$\frac{1}{x \cdot x}$	$\frac{1}{x \cdot x \cdot x}$

with the ones cubes for single digit multiplication, we have missed a valuable opportunity for showing students consistency, for using the recall value of visual organization, and for avoiding the math-phobic's nightmare of yet another confusing and unrelated ritual. Lola June May gave us an excellent series of lessons in her 1991 article in *Teaching PreK–8* magazine that showed us how to expand the regrouping notion to hundreds, tens, and ones with two-digit multiplication using the Base 10 blocks in the third and fourth grades. Also, the blocks provide an opportunity to represent area as the product and then later use the same organization to present the algebraic representation with algebra tiles.

To illustrate how the Base 10 blocks evolve into algebra tiles, I have drawn an example for multiplication with Base 10 alongside a comparable illustration using algebra tiles.

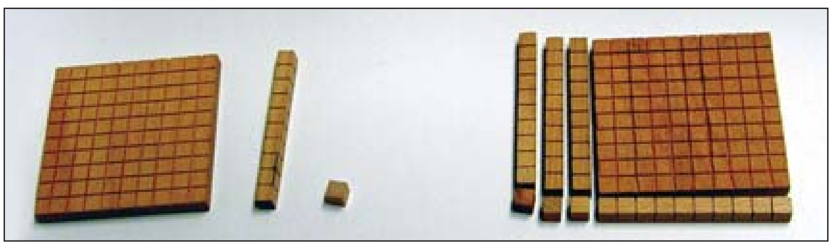
Representing the algebra multiplication process (older students know this as FOIL) with algebra tiles also allows for a connection between the "cross hatch process" for multiplying two digit numbers with quadratics along with algebra tiles.

Place value location decided by base 10

	Hundreds	Tens	Ones (unit)
		1	3
multiply		1	1
		1	3
	1	3	
	1	4	3

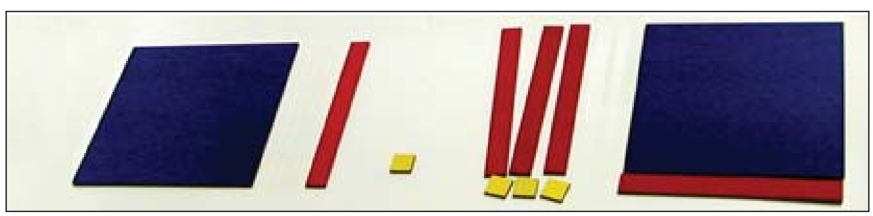
Place location decided by kind of term (also known as like terms)

	Square	Linear	unit
		x	3
multiply		x	1
		x	3
	x^2	3x	
	x^2	4x	3



hundreds	tens	ones	sides	Area as product
square	linear		3 x 11	1 hundred + 4 tens + 3 ones
10^2	10		(10+3)(10+1)	100 + 40 + 3
10 x 10	10 x 1			143

The same design for multiplying two digit (or more) numbers is used with the algebra of multiplying sides (or factors) in quadratic equations. This arithmetic connection to algebra tiles was demonstrated by Peter Rasmussen in 1978 in a teacher manual accompanied by arithmetic and algebra tiles entitled *MathTiles* and again in 1987 in *Algebra in the Concrete* by Mary Laycock and Reuben Schadler.



Square is x^2	Long is x^1	ones	sides	Area as product
Since we do not know how long the side of the square is then we need to give it an algebraic letter symbol that represents "don't know" and later expand this notion into "it doesn't matter because the system works for all numbers."	Since the length of the skinny rectangle is the same as the length of the square then we can give it the same length label as the square. x; $x \cdot 1$		(x + 3)(x + 1)	$x^2 + 3x + 1x + 3$; $x^2 + 4x + 3$. Add "like terms" because they are like shapes.

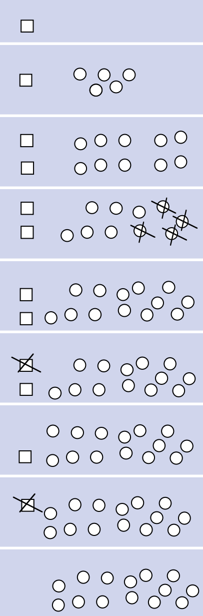
So you see, many algebra skills can be in place well before students get to Algebra I; the key lies in the consistency and in their fluid thinking, the ability so valued in problem solving – another highly regarded standard from NCTM's *Principles and Standards for School Mathematics*. If students and the teachers recognize what is "going on" with the symbolism at each level of the continuum, have a visual model to associate the skill or concept with, and recognize the consistency built into the notation then the problem solving with applications will take care of itself. The mathematics is in the consistency and the systematic thinking in the getting of the answers as well as in the thinking about the answers!

Cathy Draper has more than forty years experience in math education, starting as a high school teacher, then district supervisor, state level advisor, tutor, professional development consultant, and as an adjunct math professor for several colleges. She has also worked as a math textbook editor and math supplementary materials editor. She started The Math Studio in the early 80's and published "The Algebra Game" and "Solving with Pythagoras" for cooperative learning groups. Visit the website at www.mathstudio.com *to learn about other math and algebra connections.*

Take a Number

The age-old mental math activity that I used to play in my own elementary grades called, "Think of a Number," is easily adapted to another literacy-manipulative-symbolic activity named, "Take a Number," and applied to algebraic symbols. Use squares that are outdated counter color tiles scavenged from hardware stores and poker chips. The squares represent the known number, the one that a student thinks of but doesn't tell what it is, and the poker chips represent the number that was either added or subtracted according to the literacy directions. For the algebraic example, I also use poker chips and squares allowing the square to represent the "**x**" for the number that we made up and the poker chips for the addition or subtraction of numbers:

Use objects to show what is happening with the numbers



LITERACY MODEL	MANIPULATIVE MODEL	ALGEBRAIC EXPRESSION MODEL
Think of a number.		**x**
Add 5.		**x + 5**
Double your new number.		**2(x + 5)**
Subtract 4.		**2x + 10 – 4**
Add 7.		**2x + 6 + 7**
Subtract your first number.		**2x + 13 – x**
Add 1		**x + 13 + 1**
Subtract your first number again.		**x+14-x**
Your answer is 14.		**14**

Why does this work?

The "answer" is in the poker chips.