How I Used Classification Skills To Jump-Start My Students into the Critical Thinking Process

Catheryne Draper
The Math Studio, Inc.

Is critical thinking a desirable component in mathematics classes? Absolutely. Does classification and sorting help to develop critical thinking strategies? While my intuition has always said "yes," my recent review of some of the critical thinking literature did not provide an obvious corroborating "yes." Of the dozen articles I reviewed defining the critical thinking process, only two specifically cited classification in their definitions. My informal interviews with K-12 colleagues on this topic yielded mixed opinions regarding a direct connection although no one considered classification as a non-essential activity.

Classification is generally accepted, or should be [Skemp, 1987], as a worthwhile task in mathematics classrooms at all grade levels. Few would disagree that incorporating critical thinking strategies is one of the top priorities for any mathematics classroom, or any classroom [Willingham, 2007], at all grade levels. However, I believe it is safe to say that too many upper grade math classrooms are being pressed to "cover" a large body of information and tend to defer to algorithm process instruction rather than sense-making activities [Shoenfeld, 1993]. My question is, "Couldn't the simple act of classifying mathematical terms actually *generate* the critical thinking processes involved in sense-making activities?"

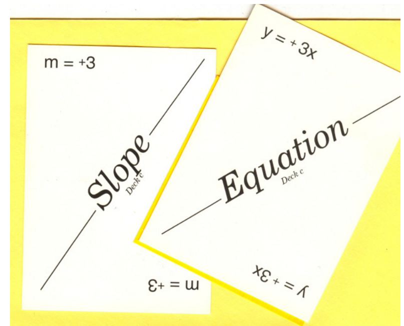
This article briefly describes how I used classification to stimulate critical thinking with my own students and how I later solicited opinions about this link between the instructional benefits of classifying and critical thinking behaviors from other math educators within a professional development program.

Getting students ready

My challenge as a classroom teacher was to provide activities that generated critical thinking and instigated active student involvement, while still maintaining the usual course content requirements. I chose classification of mathematical terms as the process to address all three concerns. Not having much success finding materials appropriate for my algebra class population (passive learners and not necessarily the stronger math students) I did what many teachers do; I made my own. I developed decks of cards using terms from both the textbook and my own lessons so that the students could sort, arrange, and rearrange these terms to first get familiar with the language and then, through further classifications, could identify relationships and conjecture about connections. In subsequent lessons, I used the cards to allow students to organize their thoughts around the related concepts. Eventually, these decks were published as **The Algebra Game** and **Solving with Pythagoras**. [These are available at Nasco Education www.enasco.com , EAI Education www.eaieducation.com , Didax Educational Resources www.didax.com, ETA/Classroom Products www.classroomproductswarehouse.com or Greene Bark Press www.greenebarkpress.com . Canadian teachers can order from Spectrum Education at www.spectrumed.com At all sites search for **Algebra Game** and **Solving with Pythagoras**.]

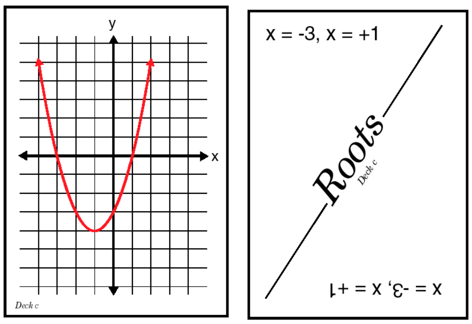
If students have been subtly trained over the years to "read the teacher's mind for the right answer," then classifying according to more open-ended or general descriptions can be a daunting task. So, initially, I eased the students into active involvement with a "Start Up" matching activity. With only one thing to decide and each other to help, my students were more open to the risk of actually talking about the algebra, sharing information, and making a decision. The general format for most of my Start Up activities was to give one half of the class one type of card from a given deck and the other half another matching card from the same deck. The task: find a partner who had the card that would match your card. My time limit was always three to five minutes, enough time for them to talk to others and find a match but not so long that frustration was evident. I wanted to state the desired outcome but I did not want to tell them how to accomplish the task, as that would reinforce their previous subtle training, that they had to read my mind to get the right answer. I chose cards that would require minimal or no prior instruction (the match was hopefully obvious to enough of the students who would then be able to tell the others) to give them an initial successful experience without being "told how to think."

One Start Up example that I used in Algebra I classes required the students to match Slope cards to Equation cards in Linear Graphs *deck c* because *deck c* does not have any distracting *y*-intercepts in the equations. The Slope card had "m = 3" and Equation card had "y = 3x" [Figure 1]; all they needed to do was visually locate the commonality of "3" on both cards. Judging from the follow up class discussion, students started to learn about the significance of a coefficient location and how pieces on one card described contents on another card, without my help. They learned from each other, reasoned with each other, and described these connections in their own words. During these discussions, I overheard variations on comments like the one from one at-risk student, "Algebra is easy when you know what you are looking for."



Matching Slope and Equation cards

Figure 1



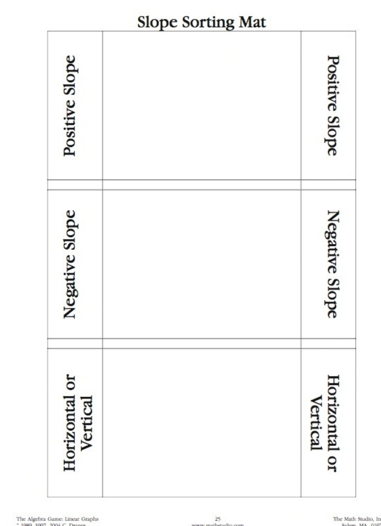
Graph and Root cards from Quadratic deck

Figure 2

In my Algebra II class, I adapted the Start Up activity for lines to Quadratics Equations by using the Graph cards and Roots cards [Figure 2] from *deck a* because *deck a* contains only parabolas with integer roots. The students were told to find their partner with the card that indicates where the parabola crosses or touches the *x*-axis. Again, success was based in a visual cue. Depending on my students' receptivity to success/risk, I would include the Point Pair card to this original mix, use the Point pair card with the Roots card, or use the Point Pair cards to match with the Graph card. The need for pre-instruction varied based on the cards selected for the matches. Sometimes the matching cards would initiate the current lesson and sometimes provide a review for the previous day's lesson.

Classification activities in my classroom

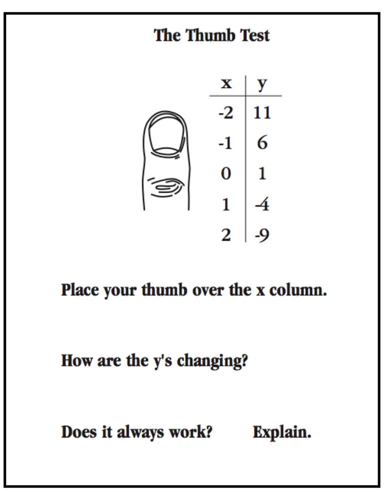
Later, as I got better at turning over responsibility for learning to my students within cooperative learning groups, these cards increasingly became more central to my lessons. I was observing not only their increased participation but also their increased and improved observation, analysis, conjecturing, all components of the critical thinking process [Warnick and Inch, 1994]. My intuition was rewarded, based on the evidence of my students' successes!



Slope Sorting Mat used with Equation cards and Slope cards.

Figure 3

I introduced my students to the classification experiences by again focusing on a visual recognition of characteristics and then gradually increasing the difficulty to require the critical thinking processes of observing, experiencing, analyzing, and applying information [Scriven and Paul, 1987]. For example, when the Slope cards were sorted on the Slope Sorting Mat, [Figure 3] students needed only to recognize the " + " and " - " sign on the card; however, when the Point Pair cards were also sorted on the same mat, students used the "Thumb Test" [Figure 4] in order to make the category determination, a decision that required pattern recognition and connecting slope to a rate of change. The concept of slope as a rate of change was problematic for my students so I used the Thumb Test to help them focus on the change pattern. Each student was asked to hold one Point Pair card, cover the *x*-coordinates with his/her thumb, and then examine the number pattern in the y column reading top to bottom. Because the *x*-coordinates were carefully controlled, the patterns were more easily recognizable as a dynamic change. At this point the

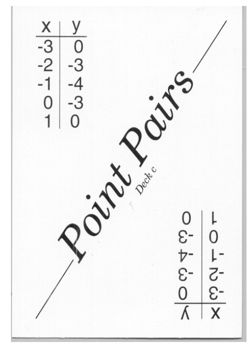


The instructional sheet used to describe The Thumb Test.

Figure 4

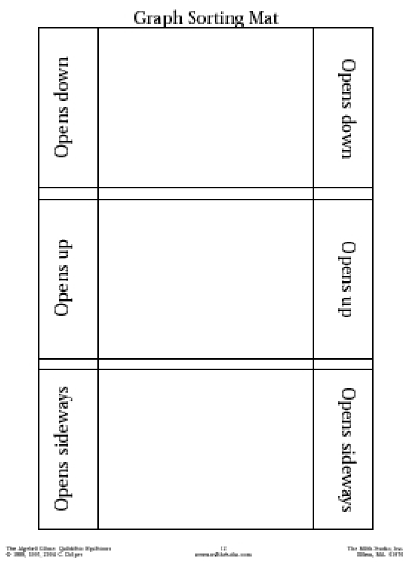
students did not know anything about slope other than it was a word on a card and that the cards had been sorted into positive and negative categories. After the Slope card and the Point Pair card sorting exercise on the same mat, I gave the students a chance to conjecture about the relationships between the Slope card and the Point Pair card and also a chance to justify their decisions. The groups offered comments that suggested increase and decrease changes, giving me an opportunity to transition to the slope formula. The students had acquired information, understood a connection, and were in the beginning stages of analyzing the information, three components of critical thinking [Reichenbach, 2000]. Later, sorting the Graph cards on the same mat provided the opportunity for students to apply the dynamic change concept to both the Point Pair card and the Graph card, algebraic and pictorial representations.

I also used the Thumb Test with Quadratic Equations so that students would observe the curve changes in the y column and eventually compare these changes with the sequences in the x column. The Point Pair card for quadratics [Figure 5] shows how the parabola moves from low to high and then back again or the reverse. Also, depending on my lesson, the *zeroes* or *roots* were obvious because of the “0” in the y column. Students sorted their Point Pair cards on the Graph Sorting Mat [Figure 6] based on their observations of the number patterns in the y column and they sorted these same cards on the Parabola Sorting Mast [Figure 7] based on the number of zeroes in the y column. In either situation, the students were becoming involved with these learning stages: Knowledge (experience, observe, and intuit), Analysis (how well we see parts/sub-parts and how these components fit together), and even an element of Synthesis (working with an amalgamation of ideas) [Anderson, Krathwohl, and Bloom, 2001].



Point Pair card from Quadratic deck

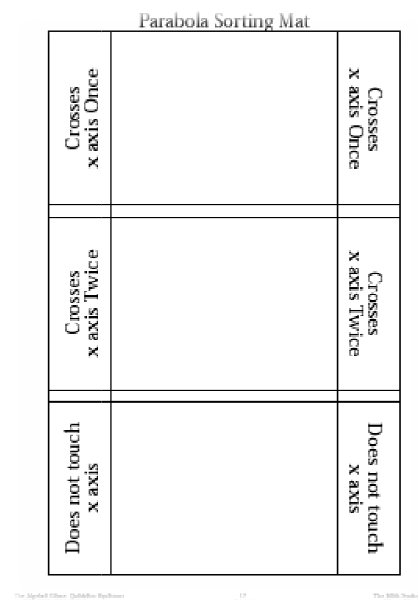
Figure 5



The Graph Sorting Mat from Quadratic Equations.

Figure 6

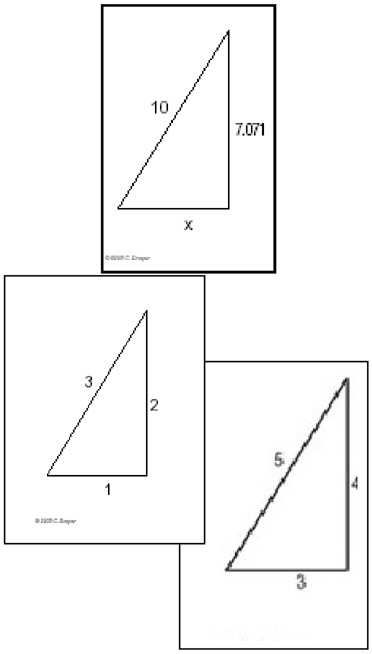
In my Geometry class, I found that a classification activity, or two, remedied a frequently occurring and seemingly automatic, but erroneous math formula application. My students could remember the Pythagorean Theorem and apply the given side values to “a” and “b” measures regardless of which sides were given in the problem setting. They seemed to be mesmerized by the rhythm of the formula $a^2 + b^2 = c^2$ and probably very relieved that they could actually remember this one! After all, they got it right at least some of the time! To make the relationship more dependable (get it right *all* of the time by recognizing the legs) and identifiable (which were legs and which was hypotenuse and how to tell the difference), I provided enough examples on cards and gave them a chance to sort the triangle cards by visual inspection (i.e. which sides are longer?) and by patterns on the sides of the triangles (i.e. which sides show half of another side?).



Parabola Sorting Mat used with Quadratic decks

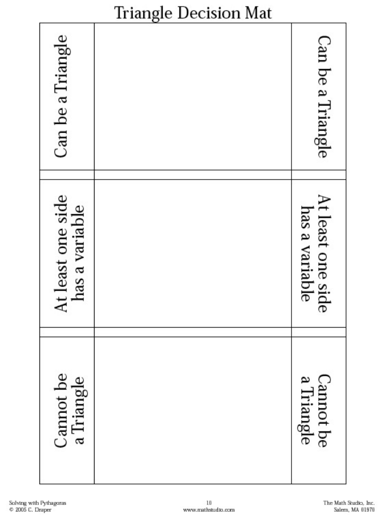
Figure 7

In order to encourage students to focus on the side measures, I first required that they sort the triangles by whether the side measurement labels permitted the triangles to be a triangle. I provided several examples of triangles with sides that would not make a triangle, such as a 1, 2, 3 side labels [Figure 8]. I wanted students to make their classification decisions based on the numerical relationships so all of the non-isosceles triangle cards looked alike and some even had variables [Figure 8]. What I did *not* expect was the heated discussion among the students about where the cards should be placed on the mat [Figure 9] - which numbers would make these triangles with variables fit into the other two sections! I allowed at least five minutes of discussion (yes, and argument) even though it was not part of my plan. Then I requested that the groups keep a record of the numbers and related triangles so that they could report their findings to the class. The students were making reasoned decisions and focusing on a desired outcome, thereby getting started with more critical thinking strategies [Halpern, 1996].



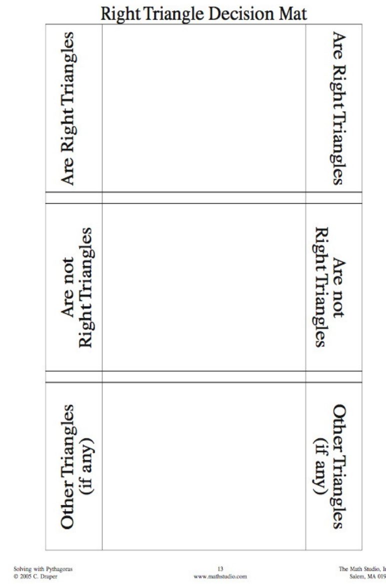
Sample triangle cards from Solving with Pythagoras

Figure 8



Triangle Sorting Mat from Solving with Pythagoras

Figure 9



Right Triangle Decision Mat from Solving with Pythagoras

Figure 10

A subsequent classification activity, using a different mat [Figure 10], allowed students to apply the Pythagorean Theorem. Students did not have to make decisions other than which numbers to use for the *a*, *b*, and *c* terms in the relationship and, because they were in groups, I let them decide, make errors, and then correct their errors. Calculators were available because my priority was on the students' experimentation rather than the calculations. When students were ready, I included the triangle cards with variables in this exercise. While sorting the right triangles, students noticed the special patterns, such as, one side is half of another in the 30º-60º-90º relationship and two equal sides in the 45º-45º-90º. Again, they noticed then inquired and eventually wrote their own expressions of these relationships, three components of critical thinking [Petress, 2004].

Testing my intuited classify/critical thinking connection in a workshop setting

My students had already experienced this classification–critical thinking connection, and now I was curious about other mathematics teachers' opinions and reactions if I put them through similar (albeit abbreviated) activities in a workshop setting. My assumption: the teachers would simulate their own students' behaviors and therefore discuss the critical thinking processes from both their students' and their own perspective. In this workshop, I asked teachers to work within cooperative learning groups to sort and classify the cards while an objective observer, another mathematics educator, tallied the critical thinking behaviors outlined on an observation sheet.

The time constraints of this workshop session required limiting the number of critical thinking components on the observation sheet; however, these selections [from Anderson, Krathewohl, and Bloom 2001; Petress, 2004; Reichenbach, 2000; Willingham, 2007; Boostrom, 2005; Gelder, Loder, Pinette, and Gelder, 2004] provided ample stimulus for discussion. Even though the observing teacher was given autonomy to make judgments interpreting behaviors, the follow-up discussion still centered more on the kinds of behaviors rather than disagreement about the tally frequency.

An example of one group's observation checklist tally is provided in the following table as an illustration of the observed behaviors. Note: tallies from a single task were applied to multiple behaviors.

Behavior identified as a component of critical thinking	Tasks and Tally of Observations			
	Linear	**Quad**	**Pythagoras**	**Totals**
Acquire information	11	3	9	**23**
Comprehend or understand	2	3	5	**10**
Apply what you understand	2	6	5	**13**
Analyze the information	6	8	7	**21**
Synthesize	5	7	6	**18**
Critically evaluate	1	4	6	**11**
Actively imagine	1	2	3	**6**
Observe keenly	5	3	3	**11**
Show facility with abstract thought	2	2	6	**10**
Sort and classify what they observe	8	9	8	**25**
Willing to submit for peer review	2	7	6	**15**
Draw conclusions from a set of facts	8	9	10	**27**
Correlate results	4	6	8	**18**
Make comparative judgments	0	4	6	**10**
Diagnose problems	1	4	4	**9**
Make decisions	3	3	6	**12**
Recognize and correct discrepancies	1	5	1	**7**

Results and conclusions from the workshop

The components that generated the largest response frequencies did not surprise the workshop participants. Their overall responses reflected an enhanced appreciation of what students must think about and do in order to classify. At the end of the three classification tasks in this workshop, the teachers generally agreed that classification activities do indeed provide an opportunity for students to get involved in critical thinking at multiple levels.

The participants were also in general agreement that the quality of the classification activity would determine the level of critical thinking required. Classification tasks that required more than one simple decision are more likely to initiate critical thinking discussion, analysis, and peer review. For example, sorting by vocabulary or visual clue only is simplistic, but requiring students to make decisions about several components requires several behaviors in the observation list: apply understanding, be willing to submit their ideas for peer review, and make comparative judgments. All workshop participants agreed that instruction in critical thinking processes required a content context rather than an attempt to teach critical thinking strategies as skills external to content.

Some summary thoughts

Throughout the elementary mathematics curriculum we are encouraged to teach our students to classify, sort, and rearrange, because in so doing our students identify relationships. In the current testing culture, with the race to improve testing scores, too many students progress through the upper grades receiving concentration on procedures at the expense of encouraging critical thinking strategies within sense-making activities. It seems to me that we teachers, especially at the upper grade levels, do not have the luxury of *not* incorporating critical thinking strategies into our instruction.

We must look to the essence of critical thinking, determine how to jumpstart it in our classrooms with *all* students. We can use cards already published or make our own to use with Classify Mats or Venn Diagrams. We can borrow ideas from the teachers in Leominster, Massachusetts, using their *Sorts* and *Splashes* downloaded from the Internet, or we can attend conference workshops and sessions to gather other strategies to adapt to our own classrooms. The NCTM Standards publication encourages us to guide students toward identifying relationships and recognizing connections. Let's do *more* of it by incorporating what we already recognize as a beneficial process, classification, into more upper-level activities to not only initiate but also perpetuate critical thinking processes.

Resources

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- Barbara Warnick and Edward Inch
- Richard Paul and Linda Elder
- Michael Skriven and Richard Paul
- Diane F. Halpern
- S. Ferrett, Lorin Anderson, David R. Krathwohl, and Benjamin S. Bloom
- Andrea Gorman-Gelder, Charlton Loder, Robert Pinette, Stuart Gelder, University of Maine

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Sorts and *Splashe*s developed by teachers in Leominster Public Schools, Leominster, MA, downloadable from University of Massachusetts Science Resource Center website www.tinyurl.com/carolhynesmath .

Warnick, Barbara and Edward S. Inch, *Critical Thinking and Communication: The Use of Reason in Argumen*t, Macmillan, 1994.

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